Given $f(x) = \frac{\sqrt{x^2+12}-2}{x^2}$ determine the following.

1) $f(2) = \frac{2}{4} = \frac{1}{2}$

2) $f(0) = \frac{\sqrt{12}-2}{0} = \text{U}$

To find out what is happening to the graph of a function when we cannot evaluate it at a certain value we use a LIMIT.

Instead of asking what does f(x) equal when x is a certain value,

we ask *what does f(x) approach as x approaches a certain value.*

Limit Notation $\lim_{x \to a} f(x) = L$

One Sided Limits

Left Hand Side: $\lim_{x \to a^-} f(x)$

Right Hand Side: $\lim_{x \to a^+} f(x)$

For a limit to exist:

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

Example 2: Find the following limit using a **table of values**:

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1} = \frac{0}{0} \quad \boxed{\frac{2}{3}}$$

First, find the left & right hand side limits... if they are equal then the limit exists & is the value you got for each.

$$\lim_{x\to 1^-}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1} =$$

x	0	1/2	.9	.99
f(x)	1	.7044	.6725	.6672

2/3

Both Agree

$$\lim_{x\to 1^+}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1} =$$

x	2	1.5	1.1	1.01
f(x)	.6275	.6439	.6614	.6661

2/3

You can also find limits by looking at the graph of a function.
Given the graph of f(x), find the following limits if they exist.

a) $\lim_{x \to 1^-} f(x) =$ 1

b) $\lim_{x \to 1^+} f(x) =$ 3

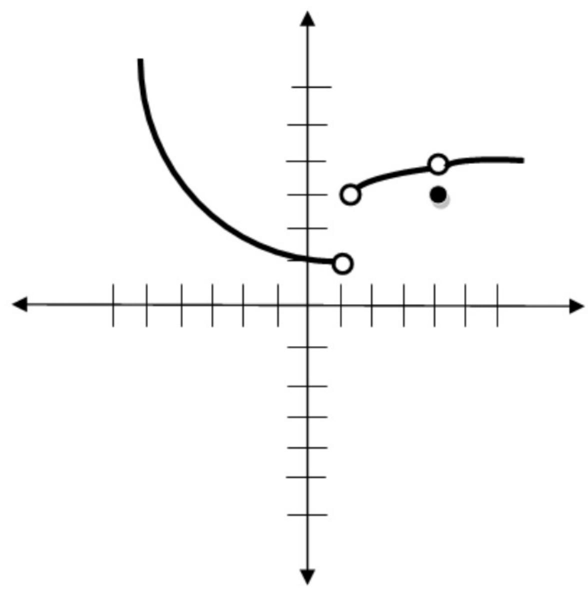
c) $\lim_{x \to 1} f(x) =$ DNE (Does not exist)

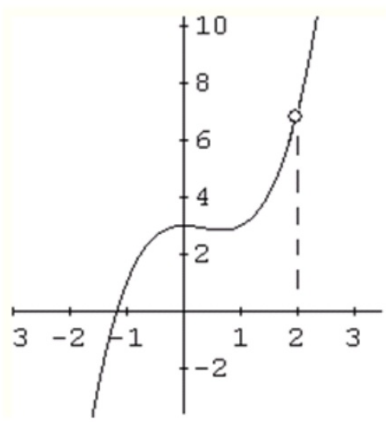
d) $\lim_{x \to 4^-} f(x) =$ 4

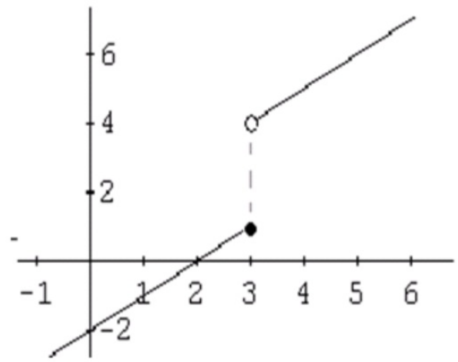
e) $\lim_{x \to 4^+} f(x) =$ 4

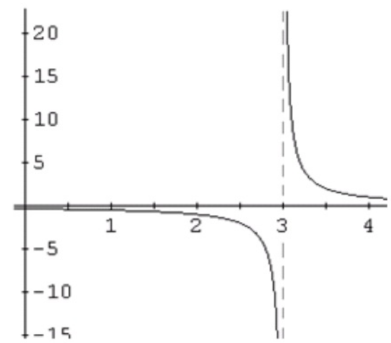
f) $\lim_{x \to 4} f(x) =$ 4

g) $f(4) =$ 3









$\lim_{x \to 2^-} f(x) = 7$

$\lim_{x \to 2^+} f(x) = 7$

$\lim_{x \to 2} f(x) = 7$

$f(2) = $ U

$\lim_{x \to 3^-} g(x) = 1$

$\lim_{x \to 3^+} g(x) = 4$

$\lim_{x \to 3} g(x) = $ DNE

$g(3) = 1$

$\lim_{x \to 3^-} h(x) = -\infty$

$\lim_{x \to 3^+} h(x) = \infty$

$\lim_{x \to 3} h(x) = $ DNE

$h(3) = $ U

Direct Substitution

If a graph is continuous you can simply plug in the value to find the limit.

1) $\lim_{x\to 2} 2x + 3x =$ $4+6$ $= \boxed{10}$

2) $\lim_{x\to -3} x^2 - 5 =$ $9-5$ $= \boxed{4}$

3) $\lim_{\theta\to \pi} \sin\theta =$ $\sin\pi = \boxed{0}$

4) $\lim_{x \to 4} \frac{3}{x+2} = \frac{3}{6} = \boxed{\frac{1}{2}}$

5) $\lim_{x \to \frac{1}{2}} (3x^2 - 2) = = \frac{3}{4} - 2$

$\boxed{-\frac{5}{4}}$

6) $\lim_{\theta \to \frac{5\pi}{4}} \tan\theta = \tan \frac{5\pi}{4}$

$= \boxed{1}$

The indeterminate form:

When direct substitution gives you $\frac{0}{0}$, simplify the function algebraically

and substitution again.

Algebraic methods for finding limits:

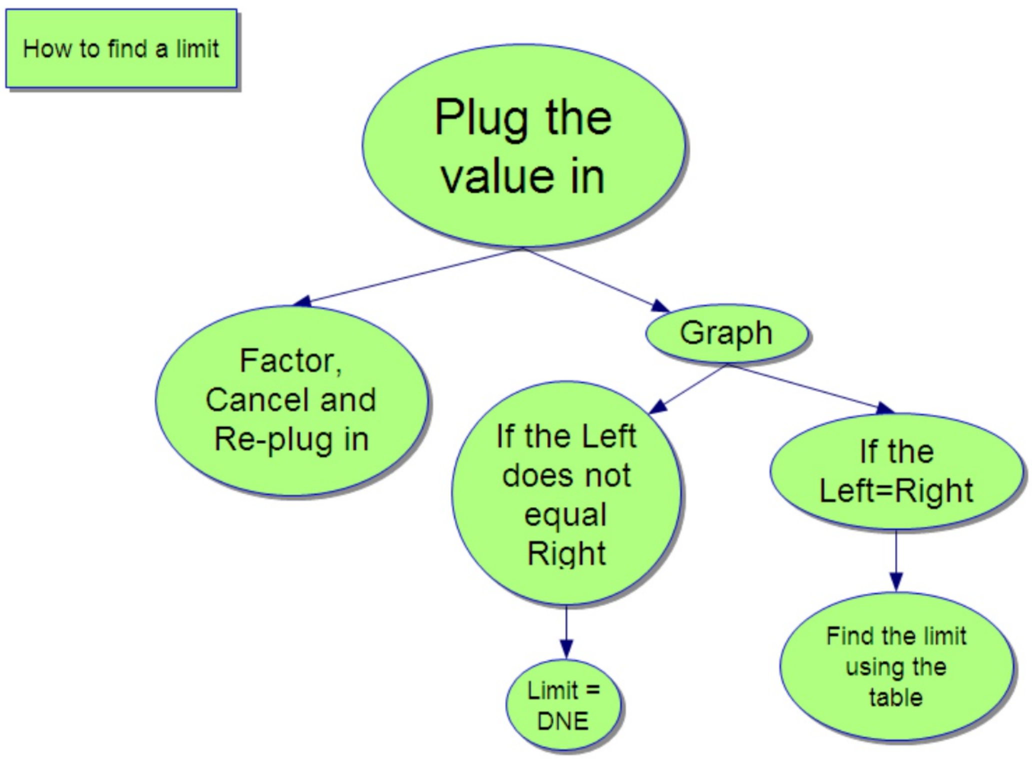
1. Direct substitution

2. Simplify expressions

7) $\lim_{x \to -3} \frac{x^2 - 9}{x + 3} =$

8) $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1} =$

9) $\lim_{x \to 4} \frac{x^2 - 7x + 12}{x^2 + 8x - 48} =$



How to find a limit
Plug the value in
Factor, Cancel and Re-plug in
Graph
If the Left does not equal Right
If the Left=Right
Limit = DNE
Find the limit using the table